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REGENERATION, PUBLIC GOODS, AND ECONOMIC GROWTH

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In this paper we discuss the (Pareto) optimal provision of a public good (desirable or undesirable) in an intergenerational model of an economy, where regeneration is an endogenous decision variable of the households in the economy. We show that in addition to providing desirable public goods and/or levying the well known Pigouvian taxes on polluting industries, a government must subsidize households who are consuming a desirable public good and tax consumers who are consuming an undesirable public good (pollution). In addition, we show that there is a public rate of return, distinct from the market rate of return, that should be used by the government for the evaluation of investments in public goods. This public rate of return is smaller than the market rate of return in a growing economy and larger than the market rate of return in a declining economy.

We investigate biases in both the public and market rates of return, as well as in other parameters which characterize the economy, as a result of nonoptimal governmental behavior. We discuss the question of how to aggregate biases due to different public goods.

1. INTRODUCTION

THE RENEWED INTEREST in the interrelationship between household behavior and economic growth has recently been emphasized by the contributions of Nerlove [**15**] and of Razin and Ben Zion [**17**], both of which offer an intergenerational model of population growth. In this paper we follow the lead of Razin and Ben Zion but add to the economy a public good (desirable or undesirable) and a government whose function is to insure the (Pareto) optimal provision of this public good. We assume a homogeneous population with respect to tastes, abilities, and initial endowments and thus we address only efficiency considerations.

In their paper, Razin and Ben Zion [**17**] also analyze the effect of governmental support for investment in children (e.g., subsidies for education), financed by personal income tax. However, the authors do not address the efficiency problem involved and the motivation for such intervention. In this paper we show that in the case of a desirable public good, such as national security, governmental intervention is required for efficient resource allocation, since a competitive system by itself will fail to achieve it. The government, in addition to supplying the public good, should also subsidize households proportionately to their size. The provision of free education in addition to national security achieves the goal of subsidizing households according to their size. This is in addition to the provision of the public good.

We also consider the case where the public good is undesirable, i.e., negative externality such as pollution. In his 1977 paper Brock[2] [**3**] introduced pollution to

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[2] For other related work, see Pitchford [**16**], Forster [**5**], and Zeckhauser, Spence, and Keeler [**20**].

the theory of optimal growth and thus expanded the theory on externalities in a static environment as presented by the leading studies of Mishan [14] in 1971 and Baumol [2] in 1972. However, he assumed a constant labor supply and population size and thus ignored an important aspect of the impact of pollution on economic growth. In our work, the focus changes to incorporate changes in population and labor supply. Moreover, the household's decision concerning its size, which is now endogenous, will also be affected by the level of pollution. And, indeed, it thus follows that besides the known requirement of levying the Pigouvian taxes on the producers, the consumer has to pay an additional corrective tax proportional to household size.

An additional result of our model is that the appropriate rate of return for evaluation of public projects (e.g., a cost benefit analysis of a public investment project) is different from the market rate of return and is usually lower in a growing economy. We also investigate possible biases in the private and public rate of returns as well as in other economic parameters, due to nonoptimal policies. We do find additional biases to those mentioned by Brock concerning the market rate of return. We also identify different biases to the public rate of return.[3]

The plan of this paper is as follows. In Section 2, the model assumptions are specified. In Section 3, the general equilibrium solution of the model is presented as an interaction of the consumer sector, producer sector, and government sector. In Section 4, the optimal solution, which provides the basis of the government behavior, is described. In Section 5, some characteristics of the solution are described, and in Section 6 the results are analyzed and a sensitivity analysis is carried out. The analysis is divided into 4 subsections. In Subsection 6.1, the market and public rates of returns are identified; in Subsection 6.2 we investigate the distortion in resource allocation in the presence of negative externalities and lack of appropriate government policy. In Subsection 6.3 we analyze distortions caused by the provision of less than the optimal amount of desirable public good or insufficient subsidization of households. We summarize the resulting biases and some policy implications in Subsection 6.4. We also develop a theory of aggregation of biases in economies with more than one public good and speculate about the estimation of those biases.

2. THE MODEL ASSUMPTIONS

Following Razin and Ben Zion [17], we assume a household which derives welfare from $C(t)$, the consumption of a composite good C at generation t, and from the welfare of the immediately following generation. Preferences are the same for each generation and can be represented by an additive utility function $V_t = U(C(t)) + \beta V_{t+1}$ where β is the subjective factor by which the current generation discounts the utility of the next generation. As Razin and Ben Zion

[3] For a summary of the directions of the biases, see Table I in Section 6.4.

[17] shows[4] this utility function leads to a household which acts as if it maximizes the function

$$V_t = \sum_{\tau=t}^{\infty} \beta^{\tau} U(C(\tau))$$

or, in a continuous form,

$$V_t = \int_t^{\infty} e^{-\delta\tau} U(C(\tau))\, d\tau,$$

where δ is the continuous counterpart of $(1-\beta)/\beta$. Now, let there be $N(t)$ identical households at generation t, and let $x(t)$ and $p(t)$ be the production (per household) of a traded composite good and a public good (per household), respectively. The public good (PG) can be either an undesirable public good (UPG) or a desirable public good (DPG), depending on its contribution to the welfare of the households in the economy.

The household's behavior in a generation t_0 is derived as if it maximizes its utility V_{t_0}. Its decisions materialize only in the present generation, t_0, while in the next generation his offspring will make their own decisions. Thus, this particular household considers its size at each time $t \geqslant t_0$ as $n(t)$, where $n(t_0) \equiv 1$; i.e., at the initial state the household perceives itself as a unit, and the rate of increase in the family size, λ, is given by

(1) $\dot{n}(t) = \lambda n(t).$

The decision making unit relevant at time t is the household at that given time, with family size assumed to be $n(t) \equiv 1$. But in its decision, the household takes into account what is expected to happen to the family in the future.

Adopting the Becker-Lancaster approach (see Michael and Becker [13]), assume that in each generation the price-taking household derives its utility from the consumption of $C(t)$, which depends on the following attributes: the quantity $x(t)$ of market goods bought by the household, the total amount of PG (desirable or undesirable) $p(t)N(t)$, the regeneration rate $\lambda(t)$, and the proportion of its time allocated to household activities τ. Thus, we obtain[5]

(2) $C = Z(x, pN, \lambda, \tau)$

(2.1) $Z_1, Z_3, Z_4 \geqslant 0, \qquad Z_2 \begin{cases} >0 & \text{iff } p \text{ is DPG,} \\ <0 & \text{iff } p \text{ is UPG,} \end{cases}$

(2.2) $Z_1(X=0) = Z_2(p=0) \quad \text{iff DPG} = Z_4(\tau=0) = \infty,$

(2.3) $Z_2(Np=\infty) \begin{cases} = -\infty & \text{iff UPG,} \\ = 0 & \text{iff DPG,} \end{cases}$

(2.4) $x, p \geqslant 0, \qquad 0 \leqslant \tau \leqslant 1, \qquad -\infty < \lambda < \infty.$

[4] Similar results in the case of intertemporal allocation of resources by the firm are obtained by Dorfman [4].

[5] From this point on whenever possible we omit the independent variable t, for reasons of simplicity of exposition.

Condition (2.1) implies positive marginal utilities from market goods, regeneration, and leisure, as well as positive marginal utility from DPG and negative marginal utility from UPG.

Condition (2.2) implies that people will never reach a state of non-consumption of market goods, DPG, and leisure. The reason is that as the stock of any of these goods is depleted an additional unit of the good will contain more and more utility. This assumption is reasonable and acceptable, as well as realistic.

Condition (2.3) is a saturation condition on the PG. On the one hand, it states that the household cannot survive if the UPG level increases indefinitely, and on the other hand it states that the household cannot live on DPG alone.

Condition (2.4) defines the range of the various variables.

On the production side, a neoclassical production function is assumed, producing an output $\hat{x}$ per household which depends on the amount of capital stock per household, k, the time input by the household, $1-\tau$, and the public good generated by the production process, p, measured per household. Hence,

(3) $\hat{x} = f(k, p, 1-\tau);$

(3.1) $f(k=0) = f(\tau = 1) = 0; \quad f(p=0)\begin{cases} =0 & \text{iff } p \text{ is UPG,} \\ \geqslant 0 & \text{iff } p \text{ is DPG,} \end{cases}$

(3.2) $f_1, f_3 > 0, \quad f_2\begin{cases} >0 & \text{iff } p \text{ is UPG,} \\ <0 & \text{iff } p \text{ is DPG,} \end{cases}$

(3.3) $f_1(k=\infty) = 0, \quad f_2(p=\infty)\begin{cases} =0 & \text{iff } p \text{ is UPG,} \\ =-\infty & \text{iff } p \text{ is DPG;} \end{cases}$

(3.4) $f_1(k=0) = f_3(\tau=1) = \infty;$

$$f_2(p=0)\begin{cases} =\infty & \text{iff } p \text{ is UPG,} \\ >-\infty & \text{iff } p \text{ is DPG.} \end{cases}$$

It should be noted that investment and market consumption goods are measured in the same units and produced by the same production process. $\hat{x}$ is the total of this composite good produced per household at time t.

Condition (3.1) states that each of the production factors is essential for the production process and we cannot do completely without any of them. The same is true with respect to the UPG, but not so with respect to the DPG. Condition (3.2) defines marginal productivities of production factors and the marginal rate of transformation between the PG and the private good produced. Condition (3.3) states the neoclassical rule of saturation for the productivity of capital and for the marginal rate of substitution between the private and the public goods. When the PG increases indefinitely its marginal product tends to approach zero if p is UPG and to approach $-\infty$ if p is DPG. The first part of (3.4) relates the neoclassical production function conditions; the second part of (3.4) states the impossibility of producing the private good without producing some quantity of UPG, but it is possible to produce the private good without producing DPG, and in order to produce some DPG a certain quantity of private good has to be sacrificed.

The distinction between UPG and DPG requires further clarification. The PG is undesirable when it causes damage to the household by reducing the output of the household production, i.e., $Z_2 < 0$. p is generated because its marginal productivity is positive in the production sector, i.e., $f_2 > 0$ and its damages are external to industry. $f(\cdot)$ in that case is a joint production function of p and $\hat{x}$, where p is a negative product.[6] A public good is desirable when the marginal productivity of the household production function $Z_2 > 0$, and the marginal opportunity cost $f_2 < 0$.[7] A typical example of a UPG is contamination of the environment by wastes or effluents generated by firms during the production process. The whole industry generates a total of Np units of pollutants which affect the population. In the case of a DPG, the signs are reversed, since the public good has a positive contribution to the utilities of the household. In this case, f represents the transformation function between the private good, x, and the collective good, p, e.g., the defense of the nation. In both cases the market mechanism fails to operate and the government may have to use policy instruments—taxes or subsidies—to be levied on households by the amount of $\mu(t)$ per unit of household size $n(t)$ and on firms $\eta(t)$ per unit of PG in order to achieve efficiency. The total amount of taxes (subsidies) collected (distributed) is then distributed (collected) to (from) the households as a lump sum. The lump sum is equally divided between the households independently of the household size.

The household's total income at time t is:

$$w + rk + g,$$

where w is the wage rate, r is the rate of capital rent, k is the amount of capital owned by the household, and g is total government transfer of income per household (positive or negative). The household spends its income on regeneration, λ, whose cost is k (we assume that capital is equally divided between household members, and that those leaving the household take their share with them); on the ratio of time spent in household production and leisure, τ, evaluated by its shadow price w; on market consumption goods, x; on tax per unit of household size, μ; and on savings $\dot{k}$, measured in units of market goods.

Accordingly the following equation of motion holds:

$$\dot{k} = w + rk + g - \lambda k - \tau w - x - \mu n. \tag{4}$$

The distinction between myopic and non-myopic decision rules in optimal capital policies depends on whether or not investment is reversible (see Arrow [**1**]). In this study the focus is on regeneration in the context of economic growth and hence the time periods are in units of generations. Thus, the irreversible decisions are made by the household according to its capital (k) and size (n). The decisions made by the firm are short-term in nature and concern the amount of capital services rented from the households and, therefore, are reversible and result in myopic decision rules. Accordingly, the industry acts as a myopic decision

[6] See Brock [**3**] and Hochman [**9, 10**] for a similar approach.

[7] See Hall [**7**] for a discussion of how transformation curves, such as f, are derived from the individual production functions.

maker and its profit function per capita is:

$$R/N = f - w(1-\tau) - rk - \sigma k - \eta p, \tag{5}$$

where σ is the rate of capital depreciation and η is the price of p paid by the industry (positive if UPG, negative if DPG) and set by the government.

It should be noted that N is not a decision variable controlled by the industry. Thus, maximizing R/N is equivalent to maximizing R.

3. THE EQUILIBRIUM SOLUTION OF THE MODEL

We are now prepared to consider the following general equilibrium problem with externalities:

(A) The representative household problem at time t_0 is to maximize

$$\int_{t_0}^{\infty} e^{-\delta t} U(c)\, dt$$

subject to (1), (2), and (4); $t_0 \leq t$.

(B) The entrepreneur problem is to maximize R/N subject to (3).

(a) *Consumer Behavior*

Let ξ_1 be the shadow price of capital, i.e., ξ_1 is the auxiliary variable associated with equation (4). ξ_2 is the shadow price of the household size, i.e., it is the auxiliary variable of equation (1). By substituting (2) for c and solving problem (A) for the necessary conditions, we obtain

$$U'Z_1 = \xi_1, \tag{6}$$

$$U'Z_3 - k\xi_1 + \xi_2 n(t) = 0, \tag{7}$$

$$U'Z_4 - w\xi_1 = 0, \tag{8}$$

$$\dot{\xi}_1/\xi_1 = \delta + \lambda(t) - r(t), \tag{9}$$

$$\dot{\xi}_2 = (\delta - \lambda)\xi_2 + \mu\xi_1. \tag{10}$$

(b) *The Production Sector*

By solving problem (B) the conditions necessary for maximization of the net gain of industry are obtained:

$$f_1 = r(t) + \sigma, \tag{11}$$

$$f_2 = \eta(t), \tag{12}$$

$$f_3 = w(t). \tag{13}$$

(c) *Interpretation of the Equilibrium Conditions*

Equation (6) translates the market price of consumption goods to utility units in which the shadow price of capital k, is measured. By dividing (8) by $U'Z_1$ and substituting (6) into the result we get

$$(14) \qquad Z_4/Z_1 = w,$$

which is the demand function for time by the household.

The demand function (15) is derived by dividing (7) by (6):

$$(15) \qquad Z_3/Z_1 = k - (\xi_2/U'Z_1)n$$

where $k-(\xi_2/U'Z_1)n$ is the total price of the regeneration rate, λ, to the household, k is the direct price, and $(-\xi_2/U'Z_1)n$ is the price (positive or negative) the household pays because of its size, where the value of the family size is evaluated through its shadow price ξ_2. In the following sections we will prove that ξ_2 is negative if μ is positive, i.e., if p is a UPG, and that ξ_2 is positive if μ is negative, i.e., if p is a DPG. When μ vanishes so does ξ_2 and $\dot{\xi}_2$ and equation (15) reduces to the simple demand function for regeneration described by Razin and Ben Zion [**17**]. In this last case, $n(t)$ ceases to be a variable.

Equations (9) and (10) describe the rate of change of the shadow prices of capital and household size. If $\alpha = \dot{\xi}_1/\xi_1$ is the rate of change of the shadow price of capital and $\beta = \dot{\xi}_2/\xi_2$, the rate of change of the shadow price of household size, then (9) can be written as

$$(16) \qquad r(t) = \delta + \lambda(t) - \alpha(t)$$

and (10), after dividing it by ξ_2 and substituting (7) and (6), can be written as

$$(17) \qquad \dot{\xi}_2/\xi_2 = \delta - \lambda - \frac{\mu n}{Z_3/Z_1 - k}.$$

By substituting β for $\dot{\xi}_2/\xi_2$ and rearranging we obtain

$$(18) \qquad Z_3/Z_1 = k + \frac{\mu n}{\delta - (\lambda(t) + \beta(t))}.$$

Equation (16) now states the known relationship that at equilibrium the rate of return of capital should equal the subjective time preference, δ, plus the rate of regeneration $\lambda(t)$ minus the rate of change in the value of capital.

Equation (18) describes the demand function for regeneration, where the price of regeneration is equal to k plus the present value of all the household's future payments of corrective taxes. It should be noted that the capitalization rate is equal to the subjective time preference δ, *minus* the regeneration rate minus the rate of change of the shadow price of population growth. It follows that there are two distinct discount rates: $r(t)$, the capital investment rate, and $\rho(t)$, defined in equation (19), the capitalization rate of investment in family size:

$$(19) \qquad \rho(t) = \delta - \lambda(t) - \beta(t).$$

Additionally, by solving (15) for $k - Z_3/Z_1$ and substituting into (18), we obtain

$$(20) \qquad \xi_2/\xi_1 = \frac{-\mu}{\delta - (\lambda + \beta)} = -\mu/\rho,$$

which shows that at equilibrium the value of the shadow price of family size (normalized by ξ_1) measured by the left hand side of (20) is equal to the discounted value of the subsidy $(-\mu)$.

The analysis carried out so far refers to a household at an arbitrary time, t_0. At time t_0, the size of the household is equal to unity and was determined at a time previous to t_0. The household decisions will be carried out only at time t_0. At any other time t, the decisions of the generation living at time t will be carried out, in accordance with the equations above. Therefore, the following relationships should be added to the above behavioral equations in order to represent the equilibrium conditions at any time t:

$$(21.a) \qquad n(t) = 1,$$

$$(21.b) \qquad \dot{N}(t) = \lambda N,$$

$$(21.c) \qquad g = \mu + \eta p.$$

Equation (21.a) is implied from the fact that the decision taken by generation t apply only to generation t, at which time $n(t) = 1$.

Equation (21.b) follows from (21.a) in the following way:

$$\dot{N} = Nd(n(t))/dt = N\dot{n} = N\lambda n = N\lambda.$$

Equation (21.c) is a government balanced budget constraint; g is positive in the case of UPG and negative when p is DPG.

The description of the equilibrium conditions of the economy is now complete. For every set of parameters, μ and η, we have a different equilibrium point. We are particularly interested in two types of equilibrium: the laissez faire case, with no government interference, where

$$(22) \qquad \eta = \mu = 0,$$

and the efficient or Pareto optimum solution case, where

$$(23) \qquad \mu = \mu_0 \neq 0; \qquad \eta = \eta_0 \neq 0.$$

4. THE OPTIMUM SOLUTION

In this section we calculate the values of μ_0 and η_0 which will result in the efficient solution of the economy. The Pareto optimum is derived from the following maximization problem:

$$\text{Max: } L = \int_0^\infty e^{-\delta t} U(c(t))\, dt$$

subject to (2), (24), and (25), where

(24) $\dot{k} = f(k, p, 1-\tau) - (\lambda + \sigma)k - x,$

(25) $\dot{N} = \lambda N.$

Let γ be the shadow price of the consumption constraint (2), and ψ_1 and ψ_2 the shadow prices of the corresponding state variables of the motion equation of capital (24), and of the number of households (25), respectively. The necessary conditions are given by equations (26)–(32):

(26) $U' = \gamma,$

(27) $\psi_1 = U'Z_1,$

(28) $f_2 = -N\dfrac{Z_2}{Z_1},$

(29) $\psi_2 = \dfrac{U'Z_1}{N}(k - Z_3/Z_1),$

(30) $f_3 = \dfrac{U'Z_4}{\psi_1},$

(31) $\dfrac{\dot{\psi}_1}{\psi_1} = \delta + \sigma + \lambda - f_1,$

(32) $\dot{\psi}_2 = \psi_2(\delta - \lambda)\psi_2 - U'pZ_2.$

By substituting (29) into (32) we obtain

(33) $\dot{\psi}_2/\psi_2 = \delta - \lambda - \dfrac{NpZ_2/Z_1}{k - Z_3/Z_1}.$

By comparing equations (26)–(33) with equations (6)–(20), we see that the equilibrium will yield an optimum iff

$$\eta = -NZ_2/Z_1 \quad \text{and} \quad \mu = -NpZ_2/Z_1.$$

That is, the optimal values of η and μ are

(34) $\eta_0 = -NZ_2/Z_1,$

(35) $\mu_0 = -NpZ_2/Z_1.$

η_0, the tax levied on the producers, is the well known Pigouvian tax discussed, for example, in Baumol and in Brock in the dynamic case. In the case of a DPG, η_0 is negative and its absolute value is equal to the price per unit the government must pay to industry for producing p. μ_0 is a per unit of household size tax payed by the population to the government in the case of a UPG and is a per unit of household size subsidy paid by the government to the population in the case of a DPG. This type of tax or subsidy is new and has not been discussed previously in the literature.

The rationale for taxing the household for a UPG it suffers from and subsidizing it for a DPG it derives benefits from is the following: The household, through regeneration, maintains its desired future size and as a result its future demand for future market goods. In the case of UPG the household demand for market goods creates a derived demand for UPG by industry. Thus, the UPG, when produced in the future, has a social cost which is external to the household when it contemplates regeneration.

By taxing population proportionally to size this external effect is internalized. When contemplating regeneration the household will take into account future costs to the household due to its current decision on regeneration. That is, the household will include the discounted value of all future taxes as part of the price of regeneration (see equation (18)). Actually current taxes do not have any influence on the household decisions, except in the sense that it serves as a predictor of the future taxes the household will have to bear.

In the case of a DPG, the subsidy encourages the household to increase its size, thus utilizing the public good p more effectively in the future and decreasing its cost per capita. Thus the subsidy actually compensates the household for increasing everybody's welfare in the future, by reducing costs of DPG in the future.

At any given time, the total tax (subsidy) collected from (paid to) industry, is equal to the total tax (subsidy) collected from (paid to) the household sector, and is equal to $(-N^2p(Z_2/Z_1))$. The total optimal lump sum tax per household is equal to

$$g=-2NpZ_2/Z_1. \tag{36}$$

g is positive if $Z_2<0$ and negative when $Z_2>0$.

5. CHARACTERIZATION OF THE SOLUTION

(a) *Laissez Faire when p is a UPG*

In this case $\eta=\mu=0$. The results obtained are the same as those obtained by Razin and Ben Zion [**17**], with addition of pollution. Thus, N ceases to be a variable and is unbounded. This may lead to disastrous results. Since $N(\infty)=\infty$ and $p(\infty)$ is positive, pollution increases with population growth, causing a decrease in utility. It follows from equation (2) that an infinite amount of pollution cannot be endured; therefore, uncontrolled growth may lead to self-destruction.

(b) *Laissez Faire with a DPG*

In this case no public good is produced and we are back in the domain investigated by Razin and Ben Zion [**17**].

(c) *The Optimum Solution—UPG*

By substituting (35) into (20) we obtain:

$$(37) \qquad \xi_2/\xi_1 = \frac{NpZ_2/Z_1}{\delta-(\lambda+\beta)}.$$

Equation (2) implies that Np will always be finite; equation (3) leads to a positive p. Hence $N(t)<\infty$ for all t, including $t=\infty$ and $\lambda(\infty)=0$. In the steady state, after substituting in (37), $\lambda(\infty)=\beta(\infty)=0$. Thus,

$$(38) \qquad \xi_2(\infty)/\xi_1(\infty) = \frac{N(\infty)p(\infty)Z_2/Z_1}{\delta} < 0.$$

It follows that $\xi_2(t)<0$ for all t since, being an auxiliary variable, it cannot change sign. From (20) we obtain an upper limit to $\lambda(t)$,

$$(39) \qquad \lambda(t) \leqslant \delta - \beta.$$

Moreover, ρ, the discount rate of family size, is now proved to be positive.

From (15) we derive

$$(40) \qquad Z_3/Z_1 \geqslant k.$$

The intuitive rationale of (40) is immediate; the price of regeneration Z_3/Z_1 that the household faces in equilibrium is now greater than k. The additional price, $\mu/(\delta-(\lambda+\beta))$, is the capitalized value of all future taxes paid by descendents of the household.

(d) *Optimal Solution—DPG*

In this case we cannot rule out any of the possibilities, $N(\infty)=\infty$ or $N(\infty)<\infty$. The value of $N(\infty)$ depends on the particular functions Z, f, and U. We will therefore try to characterize the solution under the two alternatives.

(d.1) $N(\infty)=\infty$. Under this condition, we define $\underline{P}(t)=N(t)p(t)$. Thus, $0<\underline{P}(\infty)=\underline{P}_\infty<\infty$, and $p(\infty)=0$. The expression ξ_2/ξ_1 is the discounted value of all future subsidies to the household and is therefore positive. The implication is that, since $\beta(\infty)=0$, $0<\lambda(\infty)<\delta$ in the steady state, $\lambda(t)$ is limited, i.e., $\lambda(t)<\delta-\beta(t)$. This is also implied from the transversality condition.

(d.2) $N(\infty)<\infty$. This immediately implies that $\lambda(\infty)=0$. $p(\infty)>0$, and so is $\xi_2(\infty)$ and ξ_2. Again this implies $\lambda(t)<\delta-\beta(t)$.

6. ANALYSIS OF THE RESULTS

In this section we will analyze the results of different types of deviation from the optimum and determine what kind of distortions in the optimum resource allocation result from each of them. We also look into any possible effects these deviations may have on decision parameters that may lead, in turn, to future deviations.

6.1. *Market and Public Rate of Return*

Before proceeding it should be recalled that the previous analysis contained two different discount rates: $\rho(t)=\delta-(\lambda(t)+\beta(t))$ and $r(t)=\delta+(\lambda(t)-\alpha(t))$. The public rate $\rho(t)$ is the rate of return to government investments in public goods; $r(t)$ is the rate of return to a household's investment in private markets. From the above we may deduce that

$$\rho(t)=r(t)-2\lambda(t)+(\alpha(t)-\beta(t)). \tag{41}$$

The rationale for (41) lays in the fact that $\rho(t)$ and $r(t)$ relate to two different units of the population. When a household invests in the private market it expects the yield of this investment to enable the same household to consume goods in the future, which compensates for lost present consumption.

Since the household's size changes with time at rate λ, the investment yield must be at a rate which compensates for δ, the household's time preference plus λ. In addition, the change of α, the shadow price of capital in terms of utility units, has to be taken into account. The change stems from changes in the productivity of capital due mainly to changes in the capital to labor ratio, although changes in the pollution or public goods level or changes in the amount of time spent in the labor market could also affect this shadow price. Thus, the appropriate market rate of return is $r(t)=\delta+\lambda-\alpha(t)$ where $\alpha(t)$ is the rate of change in the value of capital.

In the case of public investment the relevant household unit size does not change with time. Furthermore, since we are dealing in public goods, benefits from these goods change with changes in population size, increasing when population size increases and decreasing when population size decreases. Hence it follows that we may substitute between population size and quantity of public good. That is, in the future we would agree to a reduction in the amount of public good if there is an increase in population size. The rate of return would be the household's time preference *minus* the regeneration rate.

The change in shadow price of family size depends mainly on future taxes or subsidies the household expects to pay or get per unit household size. However, once the shadow price is different from zero, i.e., once the household expects future positive subsidies or positive taxes, other parameters may come into play. Since a change in the shadow price of household size affects our expectations from government investment in public goods, β must be included in that rate of capitalization of public investment; hence $\rho(t)=\delta-\lambda-\beta$. Even in a world without pollution or public goods, $\rho(t)$ would still be different from $r(t)$. However, in such a world there would be no need for public investment.

The question of estimating $r(t)$ and $\rho(t)$ arises. Since $r(t)$ is the market rate of return, it is readily available. By using (41) $\rho(t)$ can be estimated if we know λ, α, and β. The growth rate of the population, $\lambda(t)$, is, like $r(t)$, readily available. The problem becomes one of estimating $\alpha(t)$ and $\beta(t)$. We would consider $\alpha(t)$ to be a significant parameter mainly in periods of rapid changes in the structure of the economy (periods of major technological changes, high population migration rates, major foreign capital investment, etc.). $\beta(t)$ will be more sensitive to major changes in government policy towards the provision of public goods (desirable or not) and social programs (welfare programs, medicare, unemployment, etc.). However, beyond making an educated guess we have no idea how to estimate $\alpha - \beta$.

It is interesting to note that economists and policy makers have long felt the need for a different, usually lower, rate of return for public investment in an expanding economy.[8]

6.2 *Distortions in the Case of UPG Due to Lack of Proper Government Intervention*

The following analysis deals only with the long run effects (steady state), unless stated otherwise. Consider the case of a UPG, say pollution. Brock shows that if the firms are not charged an η equal to the social cost of pollution, the observed equilibrium market rate of interest, r_e, is likely to be larger than the socially optimum interest rate, r_0, and, at the same time, more capital will be accumulated at a faster rate. It is interesting to compare these results, where the supply of labor is assumed constant, to our model which introduces regeneration.

Let the per household tax μ_e be smaller than the optimal value (or even equal to 0). In such a case λ_e is greater than λ_0 (higher birth rate), and k_e is smaller than k_0 (lower capital per household). This can be verified by substituting $\xi_2 = 0$ in (17).

The fact that regeneration is "cheaper" than in the optimum ($\mu_e < \mu_0$) will lead to an increase in regeneration, in the supply of labor and in consumption of other goods. The increase in the consumption of x will result in an increase in pollution. On the other hand, leisure, τ, which we assume to be a complement to pollution and a substitute to market goods, decreases. The intuitive reason is that you do not enjoy your leisure if the environment is polluted and you substitute cheaper market goods for it. This means that the supply of labor per household, $1-\tau$, is increased along with the total number of labor force participants $N(t)$.

Similar effects will be obtained if $\eta_e < \eta_0$, where η is the tax per unit of pollution. Under such circumstances, pollution becomes cheaper at the laissez faire solution than at the optimum solution and is used excessively, thus increasing environmental pollution. Capital and labor become more productive, which increases the demand for them. The household then demands and uses less leisure and more market goods and regeneration to compensate for more pollution and for the increased demand for labor.

In summary, if either η or μ are smaller than their optimal values, the following distortions appear while comparing the equilibrium based on the distorted pricing of pollution with the system supported by optimal taxation:

(a) Distortion in the rate of regeneration: $\lambda_e > \lambda_0$, and in the time devoted to labor: $1-\tau_e > 1-\tau_0$.

(b) Distortion in the amount of capital per household: $k_e < k_0$. Even if there is an increase in the total amount of capital, the amount per household is decreased.

(c) Distortion in the market rate of return: $r_e > r_0$. The result obtained by Brock holds in our model as well.

[8] See, for example, Harberger [**8**] and Marglin [**12**].

(d) The distortion in ρ, the rate of return according to which public investment decisions should be made, is in the opposite direction of the distortion in $r(t)$, since $\rho(t)=\delta-\lambda(t)-\beta(t)$. In addition, $\lambda_e>\lambda_0$ and $\beta_e>\beta_0$.

Another interesting case is when $\mu_e>\mu_0$. Recently, the federal government has been striving for a situation of zero pollution. This is equivalent to letting μ_e approach infinity. We assume that $\eta_e<\eta_0$.

The increase in the cost of the UPG with respect to its optimum level tends to decrease the amount of UPG produced. This causes an initial reduction in both capital and labor productivity with respect to their optimal values. Since the price of regeneration is lower than the optimal price, λ_e, the rate of regeneration at laissez faire equilibrium is greater than λ_0. Hence when the government tries to implement a new policy of no pollution, we will initially see a decrease in the market rate of return. Since λ_e increases as well, we will see an increase in α, the rate of change of the shadow price of capital; this can be verified from (18). In the long run, however, a policy of zero pollution together with undertaxation of households will result in an increase in the rate of regeneration relative to the optimum. This in turn causes an increase in N_e, a decrease in k_e, the capital per capita, and thus leads to a long run increase in r_e, the marginal productivity of capital, and to an even greater decrease of w_e, the marginal productivity of labor, in the long run relative to the short run. Therefore, we cannot determine the sign of the bias in r_e in this case. From (18) we see that in the long run r and λ have a one to one correspondence. Thus we do not know the direction of the bias in λ as well.

The rate of return to public investment $\rho(t)$ is responsive in the long run to the same policies that $r(t)$ is, but the effect is reversed. When $r(t)$ increases, $\rho(t)$ decreases and vice versa. This can be verified by comparing (18) with (20).

6.3 *The Case of DPG—Distortions Due to Nonoptimal Government Policies*

Here we focus on the long run bias. Let us first consider the effect of insufficient finance of the public good, i.e., $\eta_e<\eta_0$. In this case more than the optimal quantities of market goods will be supplied and consumed. The regeneration rate will increase and leisure will decrease or show a smaller increase (since leisure is assumed to be a complement to the public good and a substitute to all other goods). It follows that capital per capita decreases and the equilibrium market rate of return increases relatively to the optimum. The rate of return of public investment $\rho(t)$, however, is below its optimum level. All this holds only if the subsidy to the population is at its optimal level. If the subsidy is less than optimal, it will cause a reduction in λ and thus reduce the effect of insufficient amounts of public goods. It turns out that in this case everybody will be at a lower utility level than the optimum; however, parameters characterizing the economy need not necessarily be biased, and if they are biased it cannot be predicted in which direction.

If households are subsidized above the optimal level, the bias of insufficient provision of DPG is strengthened. If public goods are provided above their optimal level, and assuming that the households are subsidized optimally, substitution effects will tend to decrease regeneration and consumption of other goods. Total income for all consumption goods except regeneration decreases due to lump sum taxes needed to finance the DPG. This reduces the consumption of x even further, but tends to increase λ, thus countering the initial substitution effect. Again we do not know what the end result will be, i.e., λ_e could be equal, smaller, or larger than λ_0.

If we do not subsidize the household sector, and assuming that the desirable public good is provided optimally, the rate of regeneration will be suboptimal, consumption of leisure and other goods will increase, rate of capital accumulation per capita will exceed the optimum as will the level of capital per capita. The wage rate will be higher than the optimum level and $r(t)$, the market rate of return, will decrease. The rate of return of public investment, $\rho(t)$, will be higher than its optimum level.

6.4 *Sensitivity Analysis*: *Summary and Some Policy Implications*

We have so far established that government policies with respect to the provision of UPG and DPG can be characterized by the level of taxes or subsidies on households, μ, and on industry, η. Each of these controls can have one of three basic positions: above, equal to, or below its optimal value.

Table I summarizes the long run biases in the market rate of return under different values of the controls η and μ. The biases of $\lambda(t)$ are the same as those of $r(t)$ (see (18)), while those of $\rho(t)$ are opposite (see (20)). The biases of $w(t)$, the wage rate, and $\tau(t)$, leisure, have no such simple relationship to $r(t)$ but since they can be determined in only a very few cases we do not deal with them in this section.

Table I shows that in the case of UPG we have more information about the direction of the biases than in the case of DPG. It can also be seen from Table I that in both UPG and DPG the biases go in both directions.

TABLE I

DIRECTION OF BIAS IN THE MARKET RATE OF RETURN

μ / η	UPG[a]			DPG[b]		
	$\mu_e < \mu_0$ (1)	$\mu_e = \mu_0$ (2)	$\mu_e > \mu_o$ (3)	$\mu_e < \mu_0$ (4)	$\mu_e = \mu_0$ (5)	$\mu_e > \mu_0$ (6)
$\eta_e < \eta_0^{(1)}$	$r_e > r_0$	$r_e > r_0$	$r_e\ ?\ r_0$	$r_e\ ?\ r_0$	$r_e > r_0$	$r_e > r_0$
$\eta_e = \eta_0^{(2)}$	$r_e > r_0$	$r_e = r_0$	$r_e < r_0$	$r_e < r_0$	$r_e = r_0$	$r_e > r_0$
$\eta_e > \eta_0^{(3)}$	$r_e\ ?\ r_0$	$r_e < r_0$	$r_e < r_0$	$r_e\ ?\ r_0$	$r_e\ ?\ r_0$	$r_e\ ?\ r_0$

[a] In the case of UPG, μ is the tax to households per unit of household size and η is tax per unit of UPG to the industry.
[b] In the case of DPG, μ is subsidy to households per unit of household size and η is a price paid to the industry per unit of DPG produced.

If we are in reality dealing with more than one public good, and assuming that we are not providing the optimal level of more than one of them, how do the biases aggregate? Let us assume that we are concerned only with marginal deviations from the optimum level of public goods. In this case deviation from the optimal level of one good is insufficient to change the conditional optimal level of the other good.[9] We already know that the biases can be described as a combination of two effects: biases due to a nonoptimal level of the public good provided and biases due to a nonoptimal behavior of households. In the production sector we have a one to one correspondence between the level of the public good produced and the amount of payments (taxes or subsidies) paid to or by industry.

The rule for aggregation of the biases is the following: if the absolute value of total payments needed to induce industry to produce given levels of a set of public goods is higher than the payments needed to induce production of the optimal levels of the same set of public goods, then the net aggregate bias of the production sector is read from Table I, row 3 ($\eta_e > \eta_0$) either in the UPG part if total payments to industry are negative or in the DPG part if total payments to industry are positive.

If the absolute value of the total payments is less than the absolute value of the optimal payments, then the aggregate bias is read from row 1 ($\eta_e < \eta_0$); if the total payments are equal ($\eta_e = \eta_0$) then the bias is read from row 2 of Table I, either in the UPG part if total payments are negative or in the DPG part if total payments are positive.

As proof of this conclusion, one should note that at the optimum, the per unit payment to industry (for producing good p_i) η_0, is equal to the marginal contribution of that unit of p_i to the utility level of the households ($-NZ_{p_i}/Z_1$) (see equation (34)) where p_i is public good i and Z_{p_i} is $\partial Z/\partial p_i$. If Δp_i is the (marginal) amount of deviation from the optimum level of p_i, the value of this deviation to the producer is $\eta_0^i \Delta p_i$ and the value to the household $-\Delta p_i(Z_{p_i}/Z_1)$. This implies that two marginal deviations of two different public goods of the same value are perfect substitutes in the sense that the household is indifferent between the two types of deviations from the optimum.[10] This implies a stronger result; the effects of these deviations can accumulate or cancel each other, depending on their respective signs. That is, the household will compensate with other goods (market goods x, leisure τ, regeneration λ) for the net aggregate value independently of the actual composition of the aggregate deviation. This implies that the net aggregate value of the deviations determines the amount of regeneration used by the household for compensation. Since λ, the rate of regeneration, is the only determining factor of the long run bias in $r(t)$ and $\rho(t)$ (see (18)), the stated conclusions follow.

It should be noted, however, that results based on the assumption of marginal deviations from the optimum may be of limited use in practice, because the deviations may be larger than marginal. Without this assumption, the results hold only if we assume a separable utility function.

When considering biases caused by nonoptimal behavior of households, no problems are encountered. Since these biases are determined by taxes and/or subsidies on household size, and since these taxes/subsidies can be added and/or subtracted, the direction of the biases can be read directly from Table I, where η_e

[9] The optimal level of the second good is conditional to the given level of the first good.

[10] By the assumption of marginal deviations only we mean that the substitution effect of the deviations can be ignored.

and η_0 are the optimal and actual aggregate net tax/subsidy per unit household, respectively. The total aggregate optimal payment to industry is positive when total aggregate payments to the household are positive and vice versa.

From observable data one can estimate the observed rate of return. There is no readily available data which will yield information about the unbiased, true rate of return. We have already noted that knowing the optimum level of public goods is, to some extent, equivalent to knowing what the optimal Pigouvian taxes (subsidies) to industry are. In addition, we have already noted that estimation of biases can be made if we know what the optimal taxes/subsidies to industry and to households are, and what the actual transfers to or from those sectors are. At present, the magnitude of the optimal transfers, as Baumol in the case of UPG and Samuelson in the case of DPG, noted, are practically impossible to determine, which implies that the biases are currently immeasurable. However, we need not end this paper with such a discouraging conclusion. Recently Groves and Ledyard [**6**], followed by Tideman and Tullock [**19**] introduced a new method for estimating the demand for a DPG. Methods for estimating the undesirability of a UPG have been known for quite some time (see, for example, Ridker and Henning [**18**] and for a more updated approach, Hochman and Miller [**11**]). These new findings bring the estimation of the optimal level of a particular public good closer to reality. Thus, we may hope that in the near future we will be able to estimate the optimal Pigouvian and household taxes and through them estimate the contribution of the biases of the particular public goods. Obviously, considerable work is still needed before this aim is achieved.

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